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How Robust Are Probabilistic Models of Higher-Level Cognition?

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Abstract

An increasingly popular theory holds that the mind should be viewed as a *near-optimal* or *rational* engine of probabilistic inference, in domains as diverse as word learning, pragmatics, naive physics, and predictions of the future. We argue that this view, often identified with Bayesian models of inference, is markedly less promising than widely believed, and is undermined by post hoc practices that merit wholesale reevaluation. We also show that the common equation between *probabilistic* and *rational* or *optimal* is not justified.

Keywords

cognition(s), Bayesian models, optimality

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Should the human mind be seen as an engine of probabilistic inference, yielding *optimal* or *near-optimal* performance, as several recent prominent articles have suggested (Frank & Goodman, 2012; Gopnik, 2012; Téglás et al., 2011; Tenenbaum, Kemp, Griffiths, & Goodman, 2011)? Tenenbaum et al. (2011) argued that “over the past decade, many aspects of higher-level cognition have been illuminated by the mathematics of Bayesian statistics” (pp. 1279–1280), pointing to treatments of language; memory; sensorimotor systems; judgments of causal strength; diagnostic and conditional reasoning; human notions of similarity, representativeness, and randomness; and predictions about the future of everyday events.

In support of this view, researchers have combined experimental data with precise, elegant models that provide remarkably good quantitative fits. For example, Xu and Tenenbaum (2007) presented a well-motivated probabilistic model “based on principles of rational statistical inference” (p. 246) that closely fit adults’ and children’s generalization of novel words to categories at different levels of abstraction (e.g., “green pepper” vs. “pepper” vs. “vegetable”) as a function of how labeled examples of those categories were distributed.

In these models, cognition is viewed as a process of drawing inferences from observed data in a fashion normatively justified by mathematical probability theory. In probability theory, this kind of inference is governed by Bayes’s law. Let D be the data and H_1 through H_k be hypotheses; assume that it is known that exactly one of the hypotheses is true. Bayes’s law states that for each hypothesis,

$$p(H_i \mid D) = \frac{p(D \mid H_i) \cdot p(H_i)}{\sum_{j=1}^{k} p(D \mid H_j) \cdot p(H_j)}$$

In this equation, $p(H_i \mid D)$ is the *posterior* probability of the hypothesis H_i given that the data D have been observed; $p(H_i)$ is the *prior* probability that H_i is true before any data have been observed; and $p(D \mid H_i)$ is the *likelihood*, the conditional probability that D would be observed assuming that H_i is true. The formula states that the posterior probability is proportional to the product of the prior probability and the likelihood. In most of the models that we discuss in this article, the “data” are information available to a human reasoner, the “priors” are a characterization of the reasoner’s initial state of knowledge, and the “hypotheses” are the conclusions that he or

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she draws. For example, in a word-learning task, the data could be observations of language, and a hypothesis could be a conclusion that the word *dog* denotes a particular category of object (friendly, furry animals that bark).

Couching their theory in the language of evolution and adaptation, Tenenbaum et al. (2011) argued that “the Bayesian approach [offers] a framework for understanding why the mind works the way it does, in terms of rational inference adapted to the structure of real-world environments” (p. 1285).

To date, these models have been criticized only rarely (Bowers & Davis, 2012; Eberhardt & Danks, 2011; Jones & Love, 2011). Here, through a series of detailed case studies, we demonstrate that two closely related problems—one of task selection, the other of model selection—undermine the conclusions that have been drawn about whether cognition is in fact either optimal or driven by probabilistic inference. Furthermore, we show that multiple probabilistic models (some compatible with the observed data but others not) are often potentially applicable to any given task, that published claims of fits of probabilistic models sometimes depend on post hoc choices that are unprincipled, and that, in many cases, extant models depend on assumptions that are empirically false, nonoptimal, or both.

Task Selection

In a recent study of physical reasoning, Battaglia, Hamrick, and Tenenbaum (in press) asked subjects to assess the stability of towers of blocks. Participants were shown a computer display of a randomly generated three-dimensional tower of blocks (for an illustration, see Fig. 1) and asked to predict whether it was stable or would fall, and if it fell, in what direction it would fall. Battaglia et al. proposed a model according to which human subjects correctly use and represent Newtonian physics, with errors arising only to the extent that subjects are affected by perceptual noise, in which the perceived *x*- and *y*-coordinates of a block vary around the actual position according to a Gaussian distribution. Within the set of problems studied, the model closely predicted the data, and the authors concluded, “Intuitive physical judgments can be viewed as a form of probabilistic inference over the principles of Newtonian mechanics” (p. 5).

The trouble with such claims is that human cognition often seems near-normative in some circumstances but not others. A substantial literature, for example, has already documented humans’ difficulties with respect to other Newtonian problems (McCloskey, 1983). For example, subjects in one study (Caramazza, McCloskey, & Green, 1981) were asked to predict what would happen if someone were swinging a rock on a string and then released the string (see Fig. 1). Most subjects predicted incorrectly that the rock would follow a circular or spiral path, rather than that the trajectory of the rock would be the tangent line. Taken literally, the conjecture of Battaglia et al. (in press) indicates that subjects should be able to answer this problem correctly; it also overestimates subjects’ ability to predict accurately the behavior of gyroscopes, coupled pendulums, and cometary orbits.

As a less challenging test of the generalizability of the probabilistic-Newtonian approach endorsed by Battaglia et al. (in press), we applied their model to balance-beam problems (for an illustration, see Fig. 1). These involve exactly the same physical principles as the tower-of-blocks problems; therefore, if Battaglia et al. were correct, it should be possible to account for subjects’ errors in terms of perceptual uncertainty. We applied their model (Gaussian distribution) of uncertainty to positional and mass information, both separately and combined. For a wide range of configurations, given any reasonable measure of uncertainty, the model predicted that subjects

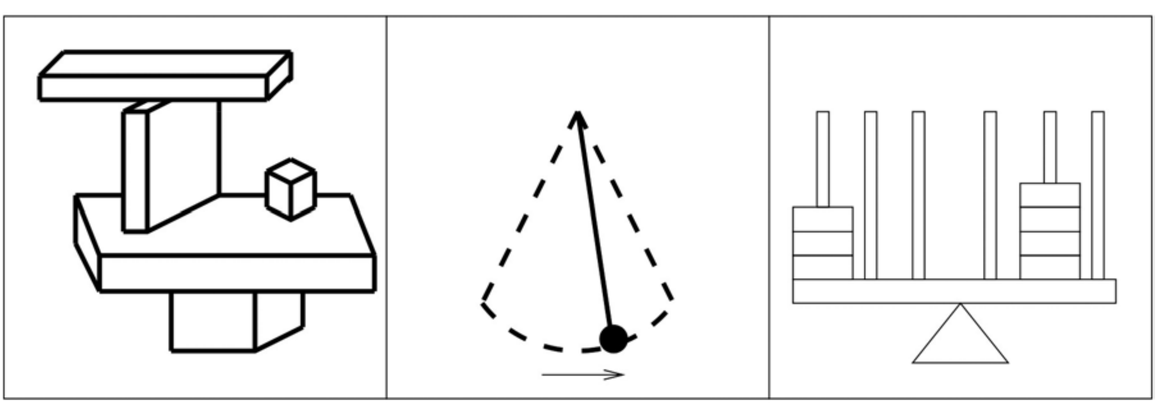

Fig. 1. Illustration of three tests of intuitive physics (from left to right): estimating the stability of a tower of blocks, estimating the trajectory that a projectile on a string will follow if released, and estimating which way a balance beam will tip. Human subjects do well on the first task, but not the other two.

would always answer the problem correctly (see the Supplemental Material available online).

As is well documented in the experimental literature, however, this prediction is false. Both children and many untutored adults (Siegler, 1976) frequently make a range of errors, such as relying solely on the number of weights to the exclusion of information about how far those weights are from the fulcrum. For this type of problem, which is only slightly different from the problems posed by Battaglia et al. (in press; both hinge on factors of weight, distance, and leverage), the model proposed by Battaglia et al. has a very poor fit. What held true in the specific case of their tower problems—that human performance is near optimal—simply is not true for a problem governed by the laws of physics applied in a slightly different configuration. (Of course, the subjects in the study by Battaglia et al. were undergraduates trained at MIT, and such sophisticated subjects may do better than more typical subjects.)

The larger concern is that the probabilistic-cognition literature as a whole may disproportionately report successes, a problem akin to Rosenthal's (1979) file-drawer problem, which would lead to a distorted perception of the applicability of the approach. Table 1 summarizes many of the most influential findings in the cognitive literature on probabilistic inference and shows that, in many domains, results that fit naturally with probabilistic techniques and claims of optimality are closely paralleled by equally compelling results that do not fit so squarely. This raises important issues about the generalizability of the framework.

The risk of confirmationism is almost certainly exacerbated by the tendency of advocates of probabilistic theories of cognition (like researchers using many computational frameworks) to follow a breadth-first search strategy, in which the formalism is extended to an ever-broader range of domains (most recently, intuitive physics and intuitive psychology), rather than a depth-first strategy, in which

Table 1. Examples of Domains in Which Performance Has Been Found to Fit Naturally With Probabilistic Explanations in Some Cases but Not Others

Domain	Apparently optimal performance	Apparently nonoptimal performance
Intuitive physics	Tower problems (Battaglia, Hamrick, & Tenenbaum, in press)	Balance-scale problems (Siegler, 1976) Projectile-trajectory problems (Caramazza, McCloskey, & Green, 1981)
Incorporation of base rates	Various tasks (Frank & Goodman, 2012; Griffiths & Tenenbaum, 2006)	Base-rate neglect (Kahneman & Tversky, 1973; but see Gigerenzer & Hoffrage, 1995)
Extrapolation from small samples	Future prediction (Griffiths & Tenenbaum, 2006) Size principle (Tenenbaum & Griffiths, 2001a)	Anchoring (Tversky & Kahneman, 1974) Underfitting of exponentials (Timmers & Wagenaar, 1977) Gambler's fallacy (Tversky & Kahneman, 1974) Conjunction fallacy (Tversky & Kahneman, 1983) Estimating unique events (Khemlani, Lotstein, & Johnson-Laird, 2012)
Word learning	Using sample diversity as a cue to induction (Xu & Tenenbaum, 2007)	Using sample diversity as a cue to induction (Gutheil & Gelman, 1997) Evidence selection (Ramarajan, Vohnoutka, Kalish, & Rhodes, 2012)
Social cognition	Pragmatic reasoning (Frank & Goodman, 2012)	Attributional biases (Ross, 1977) Egocentrism (Leary & Forsyth, 1987) Behavioral prediction of children (Boseovski & Lee, 2006)
Memory	Rational analysis (Anderson & Schooler, 1991)	Eyewitness testimony (Loftus, 1996) Vulnerability to interference (Wickens, Born, & Allen, 1963)
Foraging	Animal behavior (McNamara, Green, & Olsson, 2006) Information foraging (Jacobs & Kruschke, 2011)	Probability matching (West & Stanovich, 2003)
Deductive reasoning	Deduction (Oaksford & Chater, 2009)	Deduction (Evans, 1989)
Overview	Higher-level cognition (Tenenbaum, Kemp, Griffiths, & Goodman, 2011)	Higher-level cognition (Kahneman, 2003; Marcus, 2008)

some challenging domain is explored in great detail with respect to a wide range of tasks. More revealing than picking out arbitrary tasks in new domains might be deeper exploration of domains in which large bodies of "pro" and "anti" rationality literature are juxtaposed. For example, when people extrapolate, they are sometimes remarkably accurate, as Griffiths and Tenenbaum (2006) have shown, but at other times remarkably inaccurate, as when they anchor their judgments on arbitrary and irrelevant bits of information (Tversky & Kahneman, 1974). An attempt to understand the seemingly competing mechanisms involved might be more illuminating than the current practice of identifying a small number of tasks in each domain that seem to be compatible with a probabilistic model.

Model Selection

Closely aligned with the problem of how tasks are selected is the problem of how models are selected. Each model depends heavily on the choice of probabilities, which can come from three kinds of sources:

- Real-world frequencies
- Experimental subjects' judgments
- Mathematical models, such as Gaussian distributions or information-theoretic arguments

Moreover, a number of other parameters must also be set by basing the model or its parameters on real-world statistics either for the problem under consideration or for some analogous problem; by basing the model or its parameters on some other psychological experiment; by choosing the model or tuning the parameters to best fit the experiment at hand; or by using purely theoretical considerations, which are sometimes quite arbitrary.

Unfortunately, each of these choices can be problematic. To take one example, real-world frequencies may depend very strongly on the particular data set being used, the sampling technique, or the implicit independence assumptions. For instance, Griffiths and Tenenbaum (2006) studied estimation abilities. Subjects were asked questions like "If you heard that a member of the House of Representatives had served for 15 years, what would you predict his total term in the House would be?" The authors proposed a model in which the hypotheses were the different possible total lengths of the term, the prior was the real-world distribution of the lengths of representatives' terms, and the datum was the fact that the representative's term of service was at least 15 years. The models for the other questions in this study were analogous. These models accounted very accurately for the subjects' responses to seven of the nine questions. Griffiths and Tenenbaum concluded that "everyday cognitive judgments follow the . . . optimal statistical principles" and there is "close correspondence between people's implicit probabilistic models and the statistics of the world" (p. 767).

But it is important to realize that the fit of a model to the data depends heavily on how the priors are chosen. To the extent that priors may be chosen post hoc, the true fit of a model can easily be overestimated, perhaps greatly. For instance, one of the questions in Griffiths and Tenenbaum's (2006) study was, "If your friend read you her favorite line of poetry and told you it was line [2/5/12/32/67] of a poem, what would you predict for the total length of the poem?" (p. 770). How well a model fits this datum depends on what prior is presupposed. Griffiths and Tenenbaum based their prior on the distribution of length in an online corpus of poetry. To this distribution, they applied a stochastic model motivated by Tenenbaum's "size principle" (Tenenbaum & Griffiths, 2001a): The model assumed that (a) the choice of favorite line of poetry was uniformly distributed over poems in the corpus; (b) given a particular poem, the choice of favorite line was uniformly distributed over the lines in the poem; and (c) the subjects' answer to the question was the median of the posterior distribution.

From the apparent fit, Griffiths and Tenenbaum (2006) claimed that "people's judgements for . . . poem lengths . . . were indistinguishable from optimal Bayesian predictions based on the empirical prior distributions" (p. 770). They did not report a statistical analysis, but they included a diagram illustrating the fit. However, the fit between the model and the experimental results was not in fact as close as that diagram suggested. In the diagram, the *y*-axis represented the total length of the poem, which is the question the subjects were asked. However, it requires no great knowledge of poetry to predict that a poem whose fifth line has been quoted must have at least five lines; nor will an insurance company pay much to an actuary for predicting that a man who is currently 36 years old will live to at least age 36. The *predictive* part of these tasks is to estimate how much longer the poem will continue, or how much longer the man will live. If instead the remaining length of the poem is used as the *y*-axis, as in the left-hand panel in Figure 2, though the model has some predictive value for the data, the data are by no means "indistinguishable" from the predictions of the model.

More important, the second assumption in Griffiths and Tenenbaum's (2006) stochastic model, that favorite lines are uniformly distributed throughout the length of a poem, is demonstrably false. An online data set of favorite passages of poetry (American Academy of Poets, 1997–2013) clearly reveals that favorite passages are not uniformly distributed; rather, they are generally the first or last line of a poem, and last lines are listed as favorites about twice as frequently as first lines. As illustrated in the right-hand panel of Figure 2, a model that incorporated these empirical facts would yield a very different

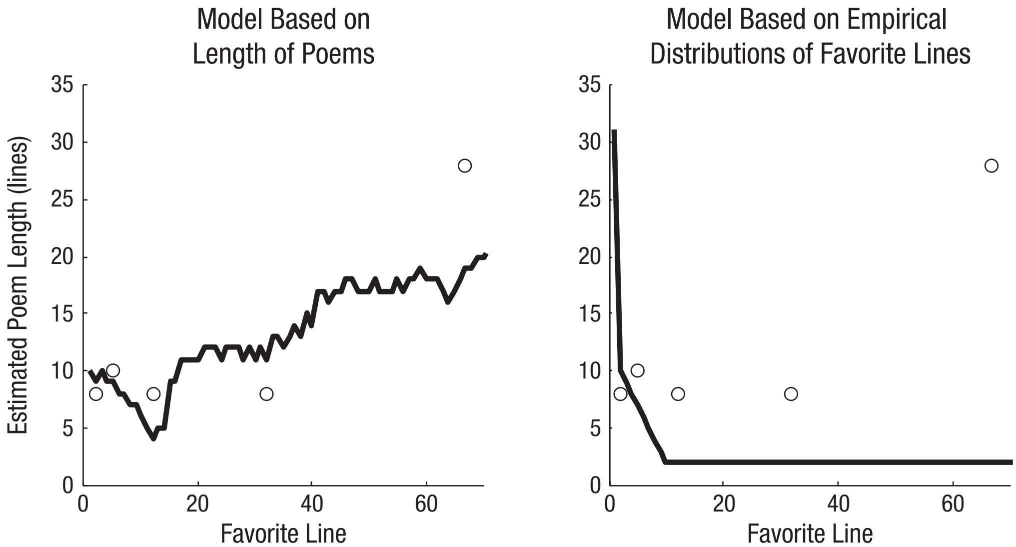

Fig. 2. Comparison of the predictions of two different probabilistic models of subjects' responses to the question, "If your friend read you her favorite line of poetry and told you it was line [2/5/12/32/67] of a poem, what would you predict for the total length of the poem?" (Griffiths & Tenenbaum, 2006, p. 770). The graph on the left shows the means of subjects' actual responses (circles) and the predictions (solid line) based on empirical data on the length of poems, combined with the assumption that the choice of favorite line was uniformly distributed over the lines in a poem. The graph on the right shows the same response data along with predictions of a model based on empirical data about distributions of favorite lines. The *x*-axes indicate the stated number of the favorite line of poetry. The *y*-axes indicate the number of lines in the poem after the chosen line, not the total number of lines in the poem.

set of predictions. Without independent data on subjects' priors, it is impossible to tell whether the Bayesian approach yields a good or a bad model, because the model's ultimate fit depends entirely on which priors subjects might actually represent. (See the Supplemental Material for a detailed discussion of the poetry data and their analysis.)

Griffiths and Tenenbaum's (2006) analysis of movies' gross earnings is likewise flawed. Subjects were asked,

> Imagine you hear about a movie that has taken in [1/6/10/40/100] million dollars at the box office, but don't know how long it has been running. What would you predict for the total amount of box office intake for that movie? (p. 770)

The data set used was a record of the gross earnings of different movies. The fit of the probabilistic model was conditioned on the assumption that movie earnings are uniformly distributed over time; for example, if a film earns a total of $100 million, the question about this movie is equally likely to be raised after it has earned $5 million, $10 million, $15 million, and so on up to $100 million. But movies, particularly blockbusters, are heavily front-loaded and earn most of their gross during the beginning of their run. No one ever heard that *The Dark Knight* (total gross = $533 million) had earned $10 million, because its gross after the first 3 days was $158 million (Wikipedia, 2013). Factoring this in would have led to a different prior (one in which projected earnings would be substantially lower) and a different conclusion (that subjects overestimated future movie earnings, and that their reasoning was not optimal).

To put this another way, the posterior distribution used by Griffiths and Tenenbaum (2006) corresponds to a process in which the questioner first picks a movie at random, then picks a number between zero and the total gross, and then formulates the question. However, if instead the questioner randomly picks a movie currently playing and formulates the question in terms of the amount of money it has earned so far, then the posterior distribution of the total gross would be very different, because the front-loading of earnings means that most of the movies playing at any given moment have earned most of their final gross. Again, one cannot legitimately infer that the model is accurate without independent evidence as to subject's priors.

Different seemingly innocuous design choices can yield models with arbitrarily different predictions in other ways as well. Consider, for instance, a recent study of pragmatic reasoning and communication (Frank & Goodman, 2012), which purportedly showed that "speakers act rationally according to Bayesian decision theory" (p. 998). In the experiment, there were two separate

Fig. 3. Illustration of the set of objects used in Frank and Goodman's (2012) study. Subjects in the listener condition were asked to place a bet on which object a speaker meant if he or she used a particular word (e.g., *blue* or *circle*) to refer to one of the objects.

groups of subjects in two different conditions, called the "speaker" condition and the "listener" condition. (A third group, in the "salience" condition, is irrelevant to the discussion here; see the Supplemental Material for details.) Subjects in the listener condition were shown a set of three objects (see Fig. 3) and asked to bet on which object a speaker would mean if he or she used a particular word to refer to one of the objects (e.g., *blue* or *circle*).

Frank and Goodman (2012) showed that a probabilistic "rational actor" model of the speaker, with utility defined in terms of *surprisal* (a measure of the information gained by the hearer) could predict subjects' performance with near-perfect accuracy (Fig. 4, left panel). The trouble is, their model depended critically on the assumption that listeners believe speakers follow a decision rule according to which they choose to use a word with a probability proportional to the word's specificity. In the case of the set shown in Figure 3, *blue* has a specificity of .5, because it applies to two objects, and *circle* has a specificity of 1, because it applies to only one object; therefore, speakers who wish to specify the middle object would use *circle* two thirds of the time and *blue* one third of the time. Although this decision rule is not uncommon, Frank and Goodman might just as easily have chosen a model with a winner-take-all decision rule, following the maximum-expected-utility principle, which is the standard rule in decision theory. According to the winner-take-all rule, listeners expect speakers to always use the applicable word of greatest specificity; this would be *circle* if the middle object were intended. As Figure 4 shows, although the model with Frank and Goodman's decision rule yielded a good fit to the data, other models, which are actually more justifiable a priori, would have yielded dramatically poorer fits. Details of the analysis are given in the Supplemental Material.

Experimenters' choice of how to word the questions posed to subjects can also affect model fit. For example, rather than asking subjects which word they would be more likely to use in a given situation (which seems ecologically natural), Frank and Goodman (2012) asked subjects in the speaker condition,

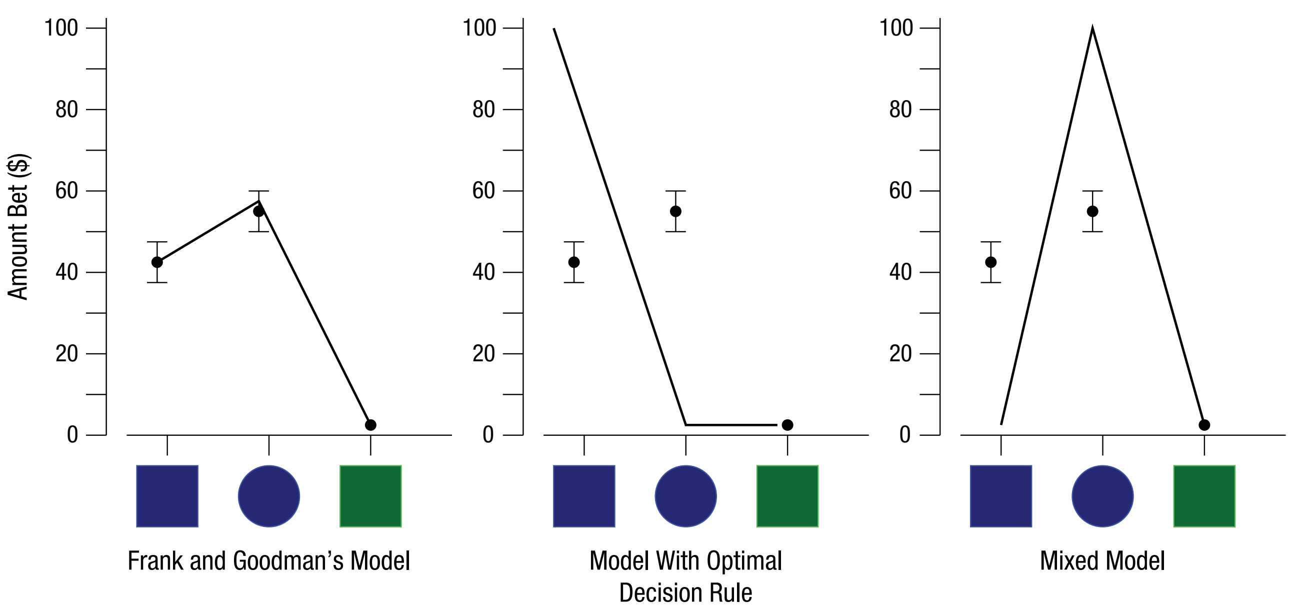

Fig. 4. Analysis of the effect of varying decision rules on the fit of probabilistic models to the data in Frank and Goodman's (2012) study. Each graph shows the mean amount subjects (listeners) bet that a speaker who used the word *blue* was referring to each of the items in the set illustrated on the *x*-axis, along with the predictions (solid lines) of a different model. The left panel shows the predictions for listeners' responses using Frank and Goodman's suboptimal model, in which listeners use a proportional decision rule and assume that speakers use a proportional decision rule. The center panel shows the predictions of a model in which listeners use a winner-take-all rule and assume that speakers use a winner-take-all rule, which is optimal. The right panel shows the predictions of a model in which listeners assume that speakers follow a proportional decision rule, but listeners follow a winner-take-all rule. As shown, the fit of the model varies considerably depending on the post hoc choice of decision rule. Error bars on the data points represent 95% confidence intervals for the empirical data.

Table 2. Features That Have Varied Across Probabilistic Models of Human Cognition

Study	Domain	Probabilities incorporated and their derivation	Decision rule
Battaglia, Hamrick, & Tenenbaum (in press)	Intuitive physics	Form of the distribution of block position: theoretically derived (Gaussian, corrected for interpenetration) Mean block position: empirically derived Standard deviation of block position: tuned post hoc	Maximum probability
Frank & Goodman (2012)	Pragmatic reasoning with respect to communication	Probability that a particular word will be chosen for a given object: derived from an information-theoretic model and confirmed by experiment Prior probability that an object will be referred to: experimentally derived	Proportional
Griffiths & Tenenbaum (2006)	Future predictions (“everyday cognition”)	Distribution of examples except waiting: empirically derived Distribution of the waiting example: derived from an inverse power law tuned to fit subjects’ responses	Median
Xu & Tenenbaum (2007)	Word learning	Priors on semantic categories and conditionals that an entity is in a category: derived from a complex model applied to experimentally derived dissimilarity judgments	Maximum probability

Note: Even in this relatively small sample of probabilistic models, model construction is based on a wide range of techniques, potentially chosen post hoc from a wider range of possible options. Many of the models in these studies would have yielded poorer fits if priors had been derived differently or if different decision rules had been invoked (see, e.g., the discussion of Frank & Goodman, 2012, in the text).

> Imagine that you have $100. You should divide your money between the possible words—the amount of money you bet on each option should correspond to how likely you would be to use that word. Bets must sum to 100! (M. C. Frank, personal communication, September 21, 2012)

In effect, subjects were asked to place a bet on what they themselves would say. The question was ecologically anomalous and coercive in that the phrasing “should divide” placed a task demand such that all-or-none-answers were pragmatically discouraged. Had subjects instead been asked, for example, whether they would use *blue* or *circle* if they were talking to someone and wanted to refer to the middle object in the set shown in Figure 3, we suspect that 100% (rather than the 67% observed) would have answered “circle” (saying “blue” would be a violation of Gricean constraints, and an actual hearer would object that this word was ambiguous or misleading).

Table 2 enumerates some of the features that have varied empirically (without a strong a priori theoretical basis) across probabilistic models of human cognition. Individual researchers are free to tinker, but the collective enterprise suffers if choices across domains and tasks are unprincipled and inconsistent. Models that have been fit only to one particular set of data have little value if their assumptions cannot be verified independently; in that case, the entire framework risks becoming an exercise in squeezing round pegs into square holes. (Another vivid example of a Bayesian cognitive model with arbitrary, debatable assumptions that came to our attention too late for inclusion here concerns infants’ use of hypotheses about sampling techniques. This study, by Gweon, Tenenbaum, & Schulz, 2011, is discussed at length in the Supplemental Material.)

At the extreme, when all other methods for explaining subjects’ errors as arising through optimal Bayesian reasoning have failed, theorists have in some cases decided that subjects were actually correctly answering a question other than the one the experimenter asked. For example Oaksford and Chater (2009) explained errors in the well-known Wason card-selection task by positing that the subjects assumed the distribution of symbols on cards that would occur in a naturalistic setting; Oaksford and Chater argued that under that assumption, subjects’ answers were in fact optimal. At first glance, this seems to offer a way of rescuing optimality, but in reality, it just shifts the locus of nonoptimality elsewhere, to the process of language comprehension.

Tenenbaum and Griffiths (2001b) adopted much the same strategy in an analysis of subjects’ expectations for a sequence of coin flips (H = heads; T = tails). Finding that subjects believed the sequence THTHTHHT is more likely than the sequence TTTTTTTT, Tenenbaum and Griffiths asserted that what subjects are trying to say was, essentially, "given THTHTHHT, the maximum likelihood hypothesis is that the coin is fair, whereas given TTTTTTT, the maximum likelihood hypothesis is that the coin is biased."

Although there may be instances in which subjects do genuinely misinterpret an experimenter's questions, such explanations should be posited infrequently and must have strong independent motivation. Otherwise, resorting to such explanations risks further weakening the predictive value of the framework as a whole. A response that can be rationalized is not the same as a response that is rational.

Discussion

Advocates of the probabilistic approach have wavered about what it is that they are showing. At some moments, they suggest that their Bayesian models are merely normative models about what humans ought to do, rather than descriptive models about what humans actually do. When the underlying mathematics is sound, there is no reason to question that modest interpretation. But there is also no reason to consider Bayesian models as null hypotheses with respect to human psychology in light of the apparent substantial empirical evidence that people sometimes deviate from normative expectations.

The real interest comes from the stronger notion that human beings might actually use the apparatus of probability theory to make their decisions, explicitly (if not consciously) representing prior probabilities, and updating their beliefs in an optimal, normatively sound fashion based on the mathematics of probability theory. It would be too strong to say that humans never behave in apparently normative fashion, but it is equally too strong to say that they always do.

As we have shown, people sometimes generalize in ways that are at odds with correctly characterized empirical data (Griffiths & Tenenbaum's, 2006, questions about poetry and films), and sometimes generalize according to decision rules that are not themselves empirically sound (Frank & Goodman's, 2012, communication task) or in ways at that are not empirically accurate (balance-beam problems). The larger literature gives many examples of each of these possibilities, ranging from the underfitting of exponentials (Timmers & Wagenaar, 1977), to probability matching (West & Stanovich, 2003), to many of the cognitive errors reviewed by psychologists such as Kahneman and Tversky (Kahneman, 2003; Tversky & Kahneman, 1974, 1983).

We have also shown that the common assumption that "performance of a Bayesian model on a task defines rational behavior for that task" (Jacobs & Kruschke, 2011, p. 9) is incorrect. As we have illustrated, there are often multiple Bayesian models that offer differing predictions because they are based on differing assumptions; at most one of them can be optimal. Even though the underlying mathematics is sound, a poorly chosen probabilistic model or decision rule can yield suboptimal results. (In three of the examples we reviewed, performance that was actually suboptimal was incorrectly characterized as optimal, in part because of an apparent match between the data and post hoc models that were Bayesian in character but incorrect in their assumptions.)

More broadly, probabilistic models have not yielded a robust account of cognition. They have not converged on a uniform architecture that is applied across tasks; rather, there is a family of different models, each depending on highly idiosyncratic assumptions tailored to an individual task. Whether or not the models can be said to fit depends on the choice of task, how decision rules are chosen, and a range of other factors. The Bayesian approach is by no means unique in being vulnerable to these criticisms, but at the same time, it cannot be considered to be a fully developed theory until these issues are addressed.

The greatest risk, we believe, is that probabilistic methods will be applied to all problems, regardless of applicability. Indeed, the approach is already well on its way to becoming a Procrustean bed into which all problems are fit, even if there are other much more suitable solutions. In some cases, the architecture seems like a natural fit. The apparatus of probability theory fits naturally with tasks that involve a random process (Téglás et al., 2011; Xu & Garcia, 2008), with many sensorimotor tasks (Körding & Wolpert, 2004; Trommershäuser, Landy, & Maloney, 2006), and with artificial-intelligence systems that involve the combination of evidence. However, in other domains, such as intuitive physics and pragmatic reasoning, there is no particular reason to invoke a probabilistic model, and it often appears that the task has been made to fit the model. It is an important job for future research to sort between cases in which the Bayesian approach might genuinely provide the best account, in a robust way, and cases in which fit depends on arbitrary assumptions.

Ultimately, the Bayesian approach should be seen as a useful tool, not a one-size-fits-all solution to all problems in cognition. Griffiths, Vul, and Sanborn's (2012) effort to incorporate performance constraints, such as memory limitations, could perhaps be seen as one step in this direction; another important step will be to develop clear criteria for what would *not* count as Bayesian performance. Another open question concerns development.

Work by Xu and Kushnir (2013) suggests that optimal, probabilistic models might be applied to children, but other studies, such as those by Gutheil and Gelman (1997) and Ramarajan, Vohnoutka, Kalish, and Rhodes (2012), suggest some circumstances in which children, too, might deviate from optimal performance.

The claims of human optimality, meanwhile, are simply untenable. Evolution does not invariably lead to solutions that are optimal (Jacob, 1977; Marcus, 2008), and optimality cannot be presumed in advance of empirical investigation. Any complete explanation of human cognition must wrestle more seriously with the fact that putative rationality very much depends on what precise task subjects are engaged in and must offer a predictive account of which tasks are and are not likely to yield normative-like performance.

More broadly, if the probabilistic approach is to make a lasting contribution to researchers' understanding of the mind, beyond merely flagging the obvious facts that people (a) are sensitive to probabilities and (b) adjust their beliefs (sometimes) in light of evidence, its practitioners must face apparently conflicting data with considerably more rigor. They must also reach a consensus on how models will be chosen, and stick to that consensus consistently. At the same time, to avoid unfalsifiability, they must consider what would constitute evidence that a probabilistic approach is not appropriate for a particular task or domain; if an endless array of model features can be varied in arbitrary ways, the framework loses all predictive value.

Author Contributions

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Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

Supplemental Material

Additional supporting information may be found at http://pss.sagepub.com/content/by/supplemental-data

References

American Academy of Poets. (1997–2013). *Life/Lines*. Retrieved from http://www.poets.org/page.php/prmID/339

Anderson, J. R., & Schooler, L. J. (1991). Reflections of the environment in memory. *Psychological Science*, *2*, 396–408.

Battaglia, P., Hamrick, J., & Tenenbaum, J. B. (in press). Simulation as an engine of physical scene understanding. *Proceedings of the National Academy of Sciences, USA*.

Boseovski, J. J., & Lee, K. (2006). Children's use of frequency information for trait categorization and behavioral prediction. *Developmental Psychology*, *42*, 500–513. doi:10.1037/0012-1649.42.3.500

Bowers, J. S., & Davis, C. J. (2012). Bayesian just-so stories in psychology and neuroscience. *Psychological Bulletin*, *138*, 389–414.

Caramazza, A., McCloskey, M., & Green, B. (1981). Naive beliefs in "sophisticated" subjects: Misconceptions about trajectories of objects. *Cognition*, *9*, 117–123.

Eberhardt, F., & Danks, D. (2011). Confirmation in the cognitive sciences: The problematic case of Bayesian models. *Minds and Machines*, *21*, 389–410.

Evans, J. S. B. T. (1989). *Bias in human reasoning: Causes and consequences*. Mahwah, NJ: Erlbaum.

Frank, M. C., & Goodman, N. D. (2012). Predicting pragmatic reasoning in language games. *Science*, *336*, 998. doi:10.1126/science.1218633

Gigerenzer, G., & Hoffrage, U. (1995). How to improve Bayesian reasoning without instruction: Frequency formats. *Psychological Review*, *102*, 684–704.

Gopnik, A. (2012). Scientific thinking in young children: Theoretical advances, empirical research, and policy implications. *Science*, *337*, 1623–1627. doi:10.1126/science.1223416

Griffiths, T. L., & Tenenbaum, J. B. (2006). Optimal predictions in everyday cognition. *Psychological Science*, *17*, 767–773. doi:10.1111/j.1467-9280.2006.01780.x

Griffiths, T. L., Vul, E., & Sanborn, A. N. (2012). Bridging levels of analysis for probabilistic models of cognition. *Current Directions in Psychological Science*, *21*, 263–268.

Gutheil, G., & Gelman, S. A. (1997). Children's use of sample size and diversity information within basic-level categories. *Journal of Experimental Child Psychology*, *64*, 159–174. doi:10.1006/jecp.1996.2344

Gweon, H., Tenenbaum, J. B., & Schulz, I. E. (2011). Infants consider both the sample and the sampling process in inductive generalization. *Proceedings of the National Academy of Sciences, USA*, *107*, 9066–9071.

Jacob, F. (1977). Evolution and tinkering. *Science*, *196*, 1161–1166.

Jacobs, R. A., & Kruschke, J. K. (2011). Bayesian learning theory applied to human cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, *2*, 8–21. doi:10.1002/wcs.80

Jones, M., & Love, B. C. (2011). Bayesian fundamentalism or enlightenment? On the explanatory status and theoretical contributions of Bayesian models of cognition [Target article and commentaries]. *Behavioral & Brain Sciences*, *34*, 169–231.

Kahneman, D. (2003). Maps of bounded rationality: Psychology for behavioral economics. *American Economic Review*, *93*, 1449–1475.

Kahneman, D., & Tversky, A. (1973). On the psychology of prediction. *Psychological Review*, *80*, 237–251.

Khemlani, S. S., Lotstein, M., & Johnson-Laird, P. (2012). The probabilities of unique events. *PLoS ONE*, *7*(10), e45975. Retrieved from http://dx.doi.org/10.1371%2Fjournal.pone.0045975

Körding, K. P., & Wolpert, D. M. (2004). Bayesian integration in sensorimotor learning. *Nature*, *427*, 244–247.

Leary, M. R., & Forsyth, D. R. (1987). Attributions of responsibility for collective endeavors. *Review of Personality and Social Psychology*, *8*, 167–188.

Loftus, E. F. (1996). *Eyewitness testimony*. Cambridge, MA: Harvard University Press.

Marcus, G. F. (2008). *Kluge: The haphazard construction of the human mind*. Boston, MA: Houghton Mifflin.

McCloskey, M. (1983). Intuitive physics. *Scientific American*, *248*(4), 114–122.

McNamara, J. M., Green, R. F., & Olsson, O. (2006). Bayes' theorem and its applications in animal behaviour. *Oikos*, *112*, 243–251.

Oaksford, M., & Chater, N. (2009). Précis of *Bayesian rationality: The probabilistic approach to human reasoning* [Target article and commentaries]. *Behavioral & Brain Sciences*, *32*, 69–120. doi:10.1017/S0140525X09000284

Ramarajan, D., Vohnoutka, R., Kalish, C., & Rhodes, M. (2012). *Does preschoolers' evidence selection support word learning?* Manuscript in preparation.

Rosenthal, R. (1979). The file drawer problem and tolerance for null results. *Psychological Bulletin*, *86*, 638–641.

Ross, L. (1977). The intuitive psychologist and his shortcomings: Distortions in the attribution process. In L. Berkowitz (Ed.), *Advances in experimental social psychology* (Vol. 10, pp. 173–220). New York, NY: Academic Press.

Siegler, R. S. (1976). Three aspects of cognitive development. *Cognitive Psychology*, *8*, 481–520.

Téglás, E., Vul, E., Girotto, V., Gonzalez, M., Tenenbaum, J. B., & Bonatti, L. L. (2011). Pure reasoning in 12-month-old infants as probabilistic inference. *Science*, *332*, 1054–1059. doi:10.1126/science.1196404

Tenenbaum, J. B., & Griffiths, T. L. (2001a). Generalization, similarity, and Bayesian inference [Target article and commentaries]. *Behavioral & Brain Sciences*, *24*, 629–791.

Tenenbaum, J. B., & Griffiths, T. L. (2001b). The rational basis of representativeness. In J. D. Moore & K. Stenning (Eds.), *Proceedings of the 23rd Annual Conference of the Cognitive Science Society* (pp. 1036–1041). Mahwah, NJ: Erlbaum.

Tenenbaum, J. B., Kemp, C., Griffiths, T. L., & Goodman, N. D. (2011). How to grow a mind: Statistics, structure, and abstraction. *Science*, *331*, 1279–1285. doi:10.1126/science.1192788

Timmers, H., & Wagenaar, W. A. (1977). Inverse statistics and misperception of exponential growth. *Attention, Perception, & Psychophysics*, *21*, 558–562.

Trommershäuser, J., Landy, M. S., & Maloney, L. T. (2006). Humans rapidly estimate expected gain in movement planning. *Psychological Science*, *17*, 981–988.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, *185*, 1124–1131. doi:10.1126/science.185.4157.1124

Tversky, A., & Kahneman, D. (1983). Extensional versus intuitive reasoning: The conjunction fallacy in probability judgment. *Psychological Review*, *90*, 293–315.

West, R. F., & Stanovich, K. E. (2003). Is probability matching smart? Associations between probabilistic choices and cognitive ability. *Memory & Cognition*, *31*, 243–251.

Wickens, D. D., Born, D. G., & Allen, C. K. (1963). Proactive inhibition and item similarity in short-term memory. *Journal of Verbal Learning and Verbal Behavior*, *2*, 440–445.

Wikipedia. (2013). *The Dark Knight* (film). Retrieved from http://en.wikipedia.org/wiki/The_Dark_Knight_(film)

Xu, F., & Garcia, V. (2008). Intuitive statistics by 8-month-old infants. *Proceedings of the National Academy of Sciences, USA*, *105*, 5012–5015. doi:10.1073/pnas.0704450105

Xu, F., & Kushnir, T. (2013). Infants are rational constructivist learners. *Current Directions in Psychological Science*, *22*, 28–32.

Xu, F., & Tenenbaum, J. B. (2007). Word learning as Bayesian inference. *Psychological Review*, *114*, 245–272. doi:10.1037/0033-295X.114.2.245