Generalized Eta and Omega Squared Statistics: Measures of Effect Size for Some Common Research Designs

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The editorial policies of several prominent educational and psychological journals require that researchers report some measure of effect size along with tests for statistical significance. In analysis of variance contexts, this requirement might be met by using eta squared or omega squared statistics. Current procedures for computing these measures of effect often do not consider the effect that design features of the study have on the size of these statistics. Because research-design features can have a large effect on the estimated proportion of explained variance, the use of partial eta or omega squared can be misleading. The present article provides formulas for computing generalized eta and omega squared statistics, which provide estimates of effect size that are comparable across a variety of research designs.

It is often argued that researchers can enhance the presentation of their research findings by including an effect-size measure along with a test of statistical significance. An effect-size measure is a standardized index and estimates a parameter that is independent of sample size and quantifies the magnitude of the difference between populations or the relationship between explanatory and response variables. Two broad categories of effect size are standardized mean differences and measures of association or the proportion of variance explained (Richardson, 1996). Kirk (1996) has provided a nice summary of the variety of measures used as estimators for effect size. Olejnik and Algina (2000) demonstrated the use and interpretation of many of these effect-size indices. Although many research methodologists have recommended the use of effect-size measures, others have been critical of their use and have cautioned that they may be easily misinterpreted (Fern & Monroe, 1996; Maxwell, Camp, & Arvey, 1981; O'Grady, 1982; Richardson, 1996). In particular, the research design of a study has

been identified as a potential source of confusion and misuse. For example, suppose two different researchers conduct studies to estimate the effect of the same treatment factor; one study includes a blocking factor (e.g., gender), but the other does not. If gender predicts the dependent variable, the study with the blocking factor will have the smaller within-cell variance. If, in each study, the within-cell standard deviation is used as the denominator of the standardized-meandifference effect size, the two studies will estimate different effect sizes even though the difference in population means is the same. Because of this problem, Glass, McGaw, and Smith (1981) recommended that meta-analysts ignore blocking factors in factorial designs when computing the standard deviation used as the denominator of a standardized mean difference. Using simulated data, Morris and DeShon (1997) demonstrated the magnitude of the error that is introduced if the blocking factor is not ignored when estimating the standardized mean difference. Glass et al. (1981, pp. 116-122) presented a similar recommendation regarding the estimation of the standardized mean difference obtained when using gain scores or covariance-adjusted posttest measures in a randomized groups pretest-posttest design. Similarly, Dunlap, Cortina, Vaslow, and Burke (1996) discussed matching and repeated measures designs and the importance of taking into consideration the correlation between measures when estimating the standardized mean difference. Olejnik and Algina (2000) discussed

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these issues and presented recommendations regarding the computation of the standardized mean difference in a wide variety of designs.

Design considerations are also important when the effect size is estimated using a measure of the proportion of variance explained, typically eta squared or omega squared. The effect of ignoring nested factors (Wampold & Serlin, 2000) and the importance of distinguishing between random and fixed factors (cf. Charter, 1982; Dodd & Schultz, 1973; Dwyer, 1974; Fleiss, 1969; Halderson & Glasnapp; 1972; Vaughan & Corballis, 1969) has also been discussed.

The number of factors studied in the completely randomized (CR) design is still another design issue that influences the proportion of variance explained. For example, consider the formula for eta squared in a CR design:

$$\hat{\eta}^2 = \frac{SS_{\text{Effect}}}{SS_{\text{T}}},\tag{1}$$

where SS_{Effect} is the sum of squares for the factor for which the effect size is being estimated and SS_T is the total sum of squares. In a multifactor CR design, the sum of squares due to each factor and due to multifactor interactions contributes to the total sum of squares for the data set. Therefore, if the same factor is investigated in several studies, but the number or nature of any additional factors varies across the studies, there may be nonrandom differences in SS_T for the studies. Consequently, eta squared for the factor of interest may not be comparable across studies. The same concern applies to omega squared because it is computed as the ratio of the estimated variance due to an effect to the estimated total variance.

Partial eta squared and partial omega squared have been recommended (e.g., Keppel, 1991, pp. 222–224) as solutions to the comparability problem. In a multifactor CR design, partial eta squared is computed as the ratio of the effect sum of squares to the sum of the effect sum of squares and the subjects-within-cells sum of squares ($SS_{s/Cells}$):

$$\hat{\eta}_P^2 = \frac{SS_{\text{Effect}}}{SS_{\text{Effect}} + SS_{\text{s/Cells}}}.$$

A convenient formula for calculating partial omega squared is

$$\hat{\omega}_P^2 = \frac{SS_{\text{Effect}} - df_{\text{Effect}}MS_{\text{s/Cells}}}{SS_{\text{Effect}} + (N - df_{\text{Effect}})MS_{\text{s/Cells}}}$$

(Keren & Lewis, 1979). Both the partial eta squared and the partial omega squared eliminate the influence of other factors in the design on the denominator of these statistics. The General Linear Model program in SPSS reports partial eta squared when the effect size is requested.

Cohen (1973) cautioned however that the use of partial eta squared may be inappropriate and can in fact be misleading when the design includes a blocking factor. A blocking factor in a CR factorial design has the effect of reducing the subjects-within-cells sum of squares. As a result, computing partial eta- or omega-squared statistics provides an estimate of effect size that is not comparable with effect sizes estimated in studies that do not include the blocking variable. The partial eta or omega squared statistics would provide estimates of effect size that can be much larger than the effect size estimated from a study that does not include the blocking factor. Although some might view the larger effect-size measure as a reward for a stronger research design, the increase in the effect size due to the blocking factor causes difficulty when comparing or aggregating effect sizes across studies. Thus partial eta and omega squared are subject to the same caution that Glass et al. (1981) set forth for the standardized-meandifference effect size.

Although the importance of the Glass et al. (1981) caution about choosing the appropriate standard deviation for computing the standardized mean difference has generally been recognized by metaanalysts, Cohen's caution on the use of partial eta squared has not, in general, been acknowledged. SPSS, for example, assumes that all factors in the design are manipulated factors and computes the partial eta squared statistics for each factor in the factorial design. Textbooks presenting analysis of variance (ANOVA) models (e.g., Keppel, 1991; Maxwell & Delaney, 2000; Stevens, 1999) and articles discussing and critiquing the use of effect-size measures (e.g., Fern & Monroe, 1996; Richardson, 1996) have not addressed the issue. Consequently, applied researchers who have been encouraged to report measures of effect size (Wilkinson and the Task Force on Statistical Inference, 1999) are most likely reporting inappropriate effect-size measures or an effect-size measure that cannot be compared across studies that do not include the same blocking factors. Effect-size measures quickly lose their usefulness unless they can be compared across a series of studies.

Purpose

The purpose of this article is to present generalized eta squared ($\hat{\eta}_{G}^{2}$) and omega squared ($\hat{\omega}_{G}^{2}$), which are alternatives to extant versions of eta and omega squared. The coefficients $\hat{\eta}_{G}^{2}$ and $\hat{\omega}_{G}^{2}$ are intended for designs in which there is at least one categorical independent variable. Each alternative provides an effect-size measure that is comparable across designs that are used to investigate the effect of a factor or the interaction of factors on the same population but vary in terms of use of blocking factors, covariates, or the inclusion of additional factors. That is, we seek to remove the potential confounding of an effect-size measure and the design used to investigate the effect. Fleiss (1969), in essence, had the same objective. In fact, the parameter we present in Equation 2 specializes to the parameters presented in Fleiss's Equations 7 and 13. However, we show how to apply the parameter and how to estimate the parameter for a much wider array of designs than was included in Fleiss.

Before introducing our procedures for calculating $\hat{\eta}_G^2$ and $\hat{\omega}_G^2,$ we define our effect-size parameter. This parameter is based on the point of view that the data in a study arise due to two sources of variance: manipulated factors in the study and individual differences. Individual differences are due to stable and transient characteristics of the participants as well as to the uncontrolled characteristics of the experimental setting that account for the fact that scores within the finest combination of the manipulated factors are not all equal. For example, in a study designed to investigate the efficacy of weight-training programs on strength, measures of individual strength within a treatment program might vary because of individual differences in gender, body type, and prior activity levels (stable characteristics). They may also differ due to such transitory characteristics as motivational levels and an individual's temporary health condition. Environmental factors such as differences in equipment quality or instrument calibration offer still another source of score variation among individuals who are in the same weight-training program. Research designs can differ in the degree to which sources of individual differences are estimated or controlled. Because our objective is to provide an effect-size measure that is comparable across a variety of research designs, our parameter recognizes and adjusts for differences in the number and type of manipulated factors as well as for differences in the degree to which sources of individual differences are estimated or controlled.

Our effect-size parameter is

$$\frac{\sigma_{\rm Effect}^2}{\delta \times \sigma_{\rm Effect}^2 + \sigma_{\rm Individual Differences}^2},$$
 (2)

where $\delta = 1$ if the effect involves only manipulated factors, and $\delta = 0$ if the effect involves one or more measured factors (e.g., gender, Gender × Manipulated factor). The parameter σ_{Effect}^2 is defined as in analysis of variance. For example, in a one-factor between-subjects design with *J* levels,

$$\sigma_{\text{Effect}}^2 = \frac{\sum_{j=1}^{J} (\mu_j - \mu)^2}{J},$$

where μ_i is the mean for the *j*th level of the factor, and μ is the average of the J means. The variance $\sigma^{\scriptscriptstyle {\rm \angle}}_{Individual \, Differences}$ is the sum of variance components due to measured factors, such as gender, interactions of measured factors with other factors, covariates, and variance within the cells of the design. Note that if σ_{Effect}^2 , on the one hand, is a variance component for a main effect of a measured factor, or an interaction of a measured factor with any other factors, σ_{Effect}^2 will already have been included in $\sigma^2_{Individual \, Differences}.$ Setting $\delta = 0$ simply prevents it from being included in the denominator twice. On the other hand, if σ_{Effect}^2 is a variance component for a main effect of a manipulated factor, or an interaction involving only manipulated factors, σ^2_{Effect} will not have been included in $\sigma_{\text{Individual Differences}}^2$. Setting $\delta = 1$ adds σ_{Effect}^2 to the denominator, because a manipulated factor that has an effect adds variance to the data. Several examples illustrate how the effect-size parameter and its components work in context and, in subsequent sections, we show how to estimate the parameter in a wide variety of research designs.

Suppose the effect of four allergy medicines (factor D) on psychomotor performance is investigated in a balanced between-subjects design. The effect size is

$$\frac{\sigma_{\rm D}^2}{1 \times \sigma_{\rm D}^2 + \sigma_{\rm s/Cells}^2} = \frac{\sigma_{\rm D}^2}{\sigma_{\rm D}^2 + \sigma_{\rm s/Cells}^2}.$$
 (3)

Here $\sigma_{s/Cells}^2 = \sigma_{Individual Differences}^2$ is the within-drug variance. In this example, the denominator is the total variance in the data (i.e., $\sigma_Y^2 = \sigma_D^2 + \sigma_{s/Cells}^2$), but as will become clear, this example does not imply that the denominator will always comprise the total variance.

Now suppose that participants are classified by

gender (factor G), and the two-factor design is still balanced. The effect size for drug is now

$$\frac{\sigma_{\rm D}^2}{1 \times \sigma_{\rm D}^2 + \sigma_{\rm G}^2 + \sigma_{\rm DG}^2 + \sigma_{\rm s/Cells}^2} = \frac{\sigma_{\rm D}^2}{\sigma_{\rm D}^2 + \sigma_{\rm G}^2 + \sigma_{\rm DG}^2 + \sigma_{\rm s/Cells}^2}.$$
 (4)

Here $\sigma_{s/Cells}^2$ is the variance of the criterion variable within a combination of drug and gender and does not comprise all of the variation due to individual differences. Rather, $\sigma_G^2 + \sigma_{DG}^2 + \sigma_{s/Cells}^2 = \sigma_{Individual Differences}^2$ because both the gender effect and the Gender × Drug interaction reflect the operation of individual differences. Excluding the σ_D^2 term, the denominators of Equations 3 and 4 are equal and thus estimates of the parameters defined by Equations 3 and 4 would be comparable across the two designs. That is, even though the design is different in the two examples, the magnitude of the effect size is not confounded with the design used to investigate the effect.

In addition, the design may affect $\sigma^2_{\text{Individual Differences}}$ either because of variation across experiments in control of characteristics of the experimental setting that affect the dependent variable or because of variation across experiments in the population that is sampled. As an example of variation in control of characteristics of the experimental setting, consider two designs for investigating the effect of the allergy medicines. In the first, the drugs are administered only in the morning. In the second, some participants self-select themselves into morning sessions, and some self-select into afternoon sessions. In the second study, time of day is not recorded. For both designs, the parameter defined by Equation 3 would be estimated. If there is a time-of-day effect, $\sigma_{s/Cells}^2$ will vary across designs because the second experimenter has failed to control for time of day. The parameters for the two designs will not be equal even if it happens that σ_D^2 is the same for the two designs. As an example of investigating different populations, suppose a study of the allergy drugs is conducted using only participants age 60 or older. In a second study, the participants were of majority age, but the age range was otherwise unrestricted. The effect-size parameter is again given by Equation 3 and would be larger in the study of older adults if σ_D^2 was the same in the two studies, but $\sigma_{s/Cells}^2$ was smaller in the study of older adults. In both examples, the meaning of a comparison of the effect-size parameters for the two studies would necessarily be unclear. Our measures $\hat{\eta}_G^2$ and $\hat{\omega}_G^2$ are not intended to address the comparability problem that arises when different populations are sampled, and no effect-size measure we are aware of can remove this comparability problem. Our measures are only intended to make effect-size measures comparable when different factors are manipulated in two designs, designs differ in manipulating factors as withinsubjects and between-subjects variables, and/or designs differ in the use of blocking factor or covariates. Consistent with current measures of eta and omega squared, our estimates $\hat{\eta}_{G}^{2}$ and $\hat{\omega}_{G}^{2}$, introduced below, require a balanced design. However, it may be possible to develop alternative measures for unbalanced designs by using Yates's (1934) unweighted means analysis: The cell means are treated as observations and subjected to an ANOVA (Searle, 1971).

Now suppose that the investigator who used the Drug × Gender design is interested in an effect size for gender. The effect variance is σ_G^2 , and the individual-differences variance is $\sigma_G^2 + \sigma_{DG}^2 + \sigma_{s/Cells}^2 = \sigma_{Individual Differences}^2$. Consequently the effect size is

$$\frac{\sigma_{\rm G}^2}{0 \times \sigma_{\rm G}^2 + \sigma_{\rm G}^2 + \sigma_{\rm DG}^2 + \sigma_{\rm s/Cells}^2} = \frac{\sigma_{\rm G}^2}{\sigma_{\rm G}^2 + \sigma_{\rm DG}^2 + \sigma_{\rm s/Cells}^2}$$

Note that when the effect is a measured factor, the variance due to that factor enters the denominator as component of $\sigma_{\text{Individual Differences}}^2$; its influence on the denominator through σ_{Effect}^2 is removed because $\delta = 0$.

Eta Squared

To obtain effect sizes that are comparable across designs, we propose

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm Effect}}{\delta \times SS_{\rm Effect} + \sum_{\rm Meas} SS_{\rm Meas} + \sum_{\kappa} SS_{\kappa}}, \quad (5)$$

where $\delta = 1$ if the effect of interest is a manipulated factor and zero otherwise. The index Meas runs over all sources of variance that do not include subjects but do involve a measured factors, (e.g., a blocking factor or a Block × Manipulated factor interaction), and SS_{Meas} is the sum of squares for such an effect. The index κ runs over all sources of variance that involve subjects or covariates, and SS_{κ} is the sum of squares for such a source of variation. As before, SS_{Effect} is the sum of squares for the factor for which the effect size is being estimated (see Keppel, 1991, or Kirk, 1995, for formulas to compute sum of squares).

Between-Subjects Design With Crossed Factors

In a between-subjects design with crossed factors, the only source of variance that involves subjects is subjects-within cells, so

$$\sum_{\kappa} SS_{\kappa} = SS_{s/Cells}.$$

In a CR design, all factors are manipulated factors and, therefore, $\delta = 1$ for all effects, and

$$\sum_{\text{Meas}} SS_{\text{Meas}} = 0.$$

As a result, generalized eta squared is equal to partial eta squared for a multifactor CR design. Consider a two-factor design where factor A is manipulated and factor b is a measured factor (here, we use lowercase letters to represent a measured factor such as gender). If an effect size for factor A is of interest, then SS_{Effect} = SS_A , $\delta = 1$,

$$\sum_{\text{Meas}} SS_{\text{Meas}} = SS_{\text{b}} + SS_{\text{Ab}}$$

and generalized eta squared would be computed by using

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm A}}{SS_{\rm A} + SS_{\rm b} + SS_{\rm Ab} + SS_{\rm s/Cells}} = \frac{SS_{\rm A}}{SS_{\rm Total}}$$

If an effect size for the measured factor is of interest, then $SS_{Effect} = SS_{b}$, $\delta = 0$,

$$\sum_{\text{Meas}} SS_{\text{Meas}} = SS_{\text{b}} + SS_{\text{Ab}},$$

and

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm b}}{SS_{\rm b} + SS_{\rm Ab} + SS_{\rm s/Cells}} = \frac{SS_{\rm b}}{SS_{\rm Total} - SS_{\rm A}}.$$

 SS_{a}/SS_{T}

Table 1

abc

Selected Formulas for Eta Squared Statistics in a Three-Factor Analysis of Variance

Selected Formatias for Eld Squared Statistics in a Three Fuelor Interfield Statistics				
Design ^a	Source			
	α	αβ	αβγ	
ABC	$SS_{\rm A}/(SS_{\rm A} + SS_{\rm s/ABC})$	$SS_{AB}/(SS_{AB} + SS_{s/ABC})$	$SS_{ABC}/(SS_{ABC} + SS_{s/ABC})$	
ABc	$SS_{\rm A}/(SS_{\rm T} - SS_{\rm B} - SS_{\rm AB})$	$SS_{AB}/(SS_T - SS_A - SS_B)$	$SS_{ABc}/(SS_T - SS_A - SS_B - SS_{AB})$	
AbC	$SS_{\rm A}/(SS_{\rm T} - SS_{\rm C} - SS_{\rm AC})$	$SS_{Ab}/(SS_{T} - SS_{A} - SS_{C} - SS_{AC})$	$SS_{AbC}/(SS_{T} - SS_{A} - SS_{C} - SS_{AC})$	
Abc	SS_A/SS_T	$SS_{Ab}/(SS_{T} - SS_{A})$	$SS_{Abc}/(SS_{T} - SS_{A})$	
aBC	$SS_a/(SS_T - SS_B - SS_C - SS_{BC})$	$SS_{aB}/(SS_T - SS_B - SS_C - SS_{BC})$	$SS_{aBC}/(SS_T - SS_B - SS_C - SS_{BC})$	
abC	$SS_{\rm a}/(SS_{\rm T}-SS_{\rm C})$	$SS_{ab}/(SS_{T} - SS_{C})$	$SS_{abC}/(SS_{T} - SS_{C})$	
aBc	$SS_{\rm p}/(SS_{\rm T}-SS_{\rm R})$	$SS_{aB}/(SS_{T}-SS_{B})$	$SS_{aBc}/(SS_{T}-SS_{B})$	

 SS_{ab}/SS_{T}

^a Uppercase letters indicate a manipulated factor; lowercase letters indicate a measured factor.

If the interaction between the measured factor and the manipulated factor is of interest, then $SS_{Effect} = SS_{Ab}$, $\delta = 0$,

$$\sum_{\text{Meas}} SS_{\text{Meas}} = SS_{\text{b}} + SS_{\text{Ab}},$$

and

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm Ab}}{SS_{\rm b} + SS_{\rm Ab} + SS_{\rm s/Cells}} = \frac{SS_{\rm Ab}}{SS_{\rm Total} - SS_{\rm A}}.$$

When both factors a and b are measured factors,

$$\sum_{\text{Meas}} SS_{\text{Meas}} = SS_{\text{a}} + SS_{\text{b}} + SS_{\text{ab}},$$

and generalized eta squared can be computed by using the ratio SS_{ab}/SS_{T} .

Equation 5 can be used to estimate generalized eta squared for more complicated factorial designs that include manipulated or measured factors. However, to facilitate use of generalized eta squared, Table 1 presents explicit formulas for estimating generalized eta squared in the eight possible three-factor designs involving none, one, two, or three measured factors. In Table 1, Latin letters indicate the design, and Greek letters indicate the effect of interest. To use the table, the reader labels his or her factors to be consistent with Table 1. If the goal is to calculate generalized eta squared for a main effect, the factor of interest is labeled as A if it is a manipulated factor and a if it is a measured factor. For example, in a design with an intervention, gender, and ethnic background as factors, if we are interested in an effect size for intervention, the design would be Abc, where A represents the manipulated intervention factor and b and c represent the other two factors. Generalized eta squared would equal

SS_{abc}/SS_T

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm A}}{SS_{\rm T}}.$$

If gender was the factor of interest, the design would be aBc (gender would be the a factor, intervention would be the B factor, and ethnic background would be the c factor) and

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm a}}{SS_{\rm T} - SS_{\rm B}}.$$

For a concrete example, Keppel (1991, p. 438) provided an ANOVA table for a $3 \times 2 \times 2$ factorial design where the first two factors are manipulated, and the third factor is measured (grade level). The design is ABc. Partial eta squared for the first manipulated factor would be computed as $SS_A/(SS_A + SS_{s/ABc}) =$ 33.63/(33.63 + 84.00) = .286. Generalized eta squared would be computed as $SS_A/(SS_T - SS_B - SS_B)$ SS_{AB}) = 33.63/(227.93 - 52.26 - 5.64) = .198, a 31% reduction in the effect size estimated compared with partial eta squared. The magnitude of the difference between partial eta and generalized eta squared depends on the extent to which the measured factors and the interactions between manipulated and measured factors reduce the subjects within-cell sum of squares.

Analysis of Covariance

Although completely randomized designs are very popular among applied researchers, they are inefficient. To improve the efficiency by reducing the within-cell variance, many researchers design their studies to include at least one preintervention measure that is expected to correlate with the outcome(s) of interest. Textbooks (e.g., Keppel, 1991, pp. 322-323) frequently recommend computing partial eta squared using adjusted sums of squares. However, the covariate serves a purpose similar to that of a blocking variable discussed in the previous section. Using the adjusted sum of squares to compute eta squared results in a statistic that is not comparable with estimates of effect size from studies that do not include a covariate or from studies that include a covariate with a different strength of relationship to the dependent variable. The generalized formula (see Equation 5) for eta squared can be used to provide an effect-size measure that is comparable across designs and covariates. For example, for the randomized-groups pretestposttest design with a single covariate, if eta squared for the manipulated treatment factor A was of interest then $SS_{Effect} = SS_A$, $\delta = 1$,

$$\sum_{Meas} SS_{Meas} = 0$$

because A is the only factor, and

$$\sum_{\kappa} SS_{\kappa} = SS_{\text{covariate}} + SS_{\text{s/Cells}}.$$

Then eta squared would be computed by using

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm A}}{SS_{\rm A} + SS_{\rm covariate} + SS_{\rm s/Cells}}$$

Note we are assuming a model specifying that the within-group regression slopes are equal (no Covariate \times Treatment interaction).

Repeated Measures Designs

Another approach that addresses the inefficiency of CR designs is to have all participants complete each treatment condition. These designs are referred to as repeated measures or within-subjects designs. Because observations under the conditions being studied are made on the same individuals, scores obtained under the various levels of a factor are correlated. The correlation between the measures reduces the error sum of squares used in testing hypotheses about repeated measures factors, resulting in much more efficient tests. However, as a result of the reduction in the error sum of squares, the eta squared calculated for the within-subjects design may not be comparable with the eta squared calculated if a between-subjects design was used to estimate the effects of the same factors. For example, if a one-factor between-subjects design is used to investigate the effect of factor A, then

$$\hat{\eta}^2 = \frac{SS_A}{SS_A + SS_{s/Cells}}$$

is the only eta squared that could be calculated, where $SS_{s/cells}$ estimates $(N - J) \sigma_{Individual Differences}^2$. If the A factor is investigated using a repeated measures design, a possible eta squared measure is

$$\hat{\eta}_{\rm P}^2 = \frac{SS_{\rm A}}{SS_{\rm A} + SS_{\rm sA}},$$

where SS_{sA} is the error sums of squares for the A effect. The effect sizes for the two designs are not comparable because SS_{sA} does not estimate $(N - J)\sigma_{Individual Differences}^2$ and tends to be smaller than $SS_{s/Cells}$.

To obtain eta squared for the repeated measures design that is comparable with eta squared for the between-subjects design, we should replace SS_{sA} with $SS_s + SS_{sA}$, because $SS_s + SS_{sA}$ equals the pooled sum

of squares within levels $(SS_{s/Cells})$ of the repeated measures factor (see Kirk, 1995, p. 254) and estimates $(N - J)\sigma_{Individual Differences}^2$. This replacement is exactly what happens when $\hat{\eta}_G^2$ is used. For example, if the effect size for the repeated measures factor A is of interest, $SS_{Effect} = SS_A$, $\delta = 1$,

$$\sum_{Meas} SS_{Meas} = 0,$$
$$\sum_{\kappa} SS_{\kappa} = SS_{s} + SS_{sA}$$

and

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm A}}{SS_{\rm A} + SS_{\rm s} + SS_{\rm sA}}.$$

Most computer software (e.g., SPSS, SAS) does not compute SS_s . In a one-way design two alternative methods for computing

$$\sum_{\kappa} SS_{\kappa}$$

are

$$(n-1)\sum_{j=1}^{J}S_{j}^{2},$$

where S_j^2 is the variance in the *j*th level of the factor and $SS_T - SS_A$, where SS_T is the sum of squared deviations of the observations from the grand mean.

If there are two repeated measures factors, the formulas in Table 1 can be used to estimate eta squared. The repeated measures factors are manipulated factors, and the subjects factor is a measured factor, so the design is ABc. Maxwell and Delaney (2000, p. 497) provided data for a 2×3 single-group repeated measures design. For these data, partial eta squared for the first factor would be computed as

$$\hat{\eta}_{\rm P}^2 = \frac{SS_{\rm A}}{SS_{\rm A} + SS_{\rm sA}} = \frac{289,920}{289,920 + 64,080} = .819.$$

By contrast,

$$\hat{\eta}_{G}^{2} = \frac{SS_{A}}{SS_{T} - SS_{B} - SS_{AB}}$$
$$= \frac{289,920}{1,133,940 - 285,660 - 105,120} = .390.$$

The dramatic decrease in the effect size estimated is a function of counteracting the correlation between the observations.

Mixed Designs

When a between-subjects design is combined with a repeated measures design, the result is often referred to as a *mixed design* or a *split-plot design*. A repeated measures factor, as described above, is almost never a measured factor, but a between-subjects factor may be either a manipulated or a measured factor.

Consider a study in which there is one repeated measures factor (B), there is one between-subjects factor (A), and subjects are nested within levels of factor A (s/A). If the between-subjects factor is a manipulated factor then $SS_{Effect} = SS_A$, $\delta = 1$,

$$\sum_{\text{Meas}} SS_{\text{Meas}} = 0,$$

and

$$\sum_{\kappa} SS_{\kappa} = SS_{\mathrm{S/A}} + SS_{\mathrm{Bs/A}}.$$

Generalized eta squared would then be computed as

$$\hat{\eta}_{\rm G}^2 = \frac{SS_{\rm A}}{SS_{\rm A} + SS_{\rm S/A} + SS_{\rm Bs/A}} = \frac{SS_{\rm A}}{SS_{\rm T} - SS_{\rm B} - SS_{\rm AB}}$$

Here SS_T is the sum of the squared deviations of each observation from the grand mean. This is the formula for α for design ABc (here c denotes subjects) in Table 1.

If the between-subjects factor is a measured factor (e.g., gender) and the interaction is of interest, then $SS_{Effect} = SS_{aB}, \delta = 0$,

$$\sum_{Meas} SS_{Meas} = SS_a + SS_{aB},$$
$$\sum_{\kappa} SS_{\kappa} = SS_{s/a} + SS_{Bs/a},$$

and generalized eta squared is computed by using

$$\begin{split} \hat{\eta}_{\rm G}^2 = & \frac{SS_{\rm aB}}{0 \times SS_{\rm aB} + SS_{\rm a} + SS_{\rm aB} + SS_{\rm s/a} + SS_{\rm s/a}} \\ = & \frac{SS_{\rm aB}}{SS_{\rm T} - SS_{\rm B}}, \end{split}$$

which is the eta squared formula in Table 1 for $\alpha\beta$ when the design is aBc.

Kirk (1995, p. 527) provided an ANOVA summary table for a $2 \times 2 \times 4$ mixed-model design with the last factor being within subjects. If the first betweensubjects factor is manipulated (A) and the second between-subjects factor is measured (c), the design could be described as ABc. Partial eta squared for the first between-subjects factor would be computed as $SS_A/(SS_A + SS_{s/Ac}) = 3.125/(3.125 + 7.250) = .301$. Generalized eta squared would equal $SS_A/(SS_T - SS_B - SS_{AB}) = 3.125/(235.5 - 194.5 - 19.375) = .145$.

Omega Squared

Between-Subjects Design

Although eta squared is a very popular statistic for reporting an effect size, it does provide an overestimate of the population proportion of variance explained. Peters and Van Voorhis (1940, p. 322) showed that when the explanatory and criterion variables are unrelated the bias is a function of the number of levels of the explanatory factor (J) and the total sample size, (J - 1)/(N - 1). Alternatively, Hays (1963) suggested that omega squared be used in place of eta squared. Omega squared is calculated by using unbiased estimators of the variance components associated with the sources of variation in the design. However, the ratio of the unbiased estimators is not itself an unbiased estimator of ω^2 (Winkler & Hays, 1975, p. 766). Nevertheless, omega squared tends to be less biased than eta squared in small samples (Carroll & Nordholm, 1975; Keselman, 1975).

Omega squared, however, has the same limitations as those discussed above associated with eta and partial eta squared. A generalized form for estimated omega squared that can be used with betweensubjects (e.g., completely randomized, randomized block, analysis of covariance), repeated measures, and mixed designs, when all factors except subjects are fixed, is

$$\hat{\omega}_{G}^{2} = \frac{\hat{\sigma}_{Effect}^{2}}{\delta \times \hat{\sigma}_{Effect}^{2} + \sum_{Meas} \hat{\sigma}_{Meas}^{2} + \hat{\sigma}^{2}},$$
(6)

where $\delta = 1$ if the effect of interest refers to a manipulated factor and $\delta = 0$ otherwise, the index Meas runs over all sources of variance that do not include subjects but do involve a measured factor, σ_{Meas}^2 is a variance component for a source of variance that involves a measured factor but does not involve subjects, and $\hat{\sigma}^2$ is obtained by pooling all sources of variance that involve subjects and/or covariates. A formula more convenient for calculation is

$$\hat{\omega}_{\rm G}^2 = \frac{\left(SS_{\rm Effect} - df_{\rm Effect} \times MS_{\rm Error}\right) / N}{\left[\delta\left(SS_{\rm Effect} - df_{\rm Effect} \times MS_{\rm Error}\right) + \sum_{\rm Meas}\left(SS_{\rm Meas} - df_{\rm Meas} \times MS_{\rm M_Error}\right) + N \times MS_{\rm s/Cells}\right] / N,$$
(7)

where N is the total number of scores in the analysis. We include N in the denominator and numerator of Equation 7 to indicate the connection between components of Equation 6 and components of Equation 7:

$$\hat{\sigma}_{\text{Effect}}^{2} = \frac{SS_{\text{Effect}} - df_{\text{Effect}} \times MS_{\text{Error}}}{N},$$
$$\sum_{\text{Meas}} \hat{\sigma}_{\text{Meas}}^{2} = \frac{\sum_{\text{Meas}} (SS_{\text{Meas}} - df_{\text{Meas}} \times MS_{\text{M}_\text{Error}})}{N},$$

and

$$\hat{\sigma}^2 = MS_{s/Cells}$$

To simplify the formulas that follow, N is deleted from both numerator and denominator. In $SS_{Effect} - df_{Effect} \times MS_{Error}$, the quantity MS_{Error} is the error mean square for testing the effect and in $SS_{Meas} - df_{Meas} \times MS_{M_Error}$, the quantity MS_{M_Error} is the error mean square for testing the effect labeled Meas. In fixed-effects between-subjects designs, without covariates, MS_{Error} and MS_{M_Error} are equal to $MS_{s/Cells}$. When covariates are included, MS_{Error} is equal to the within-cell variance adjusted by the covariates. In other designs $MS_{s/Cells}$ is computed from the sums of squares and degrees of freedom for several of the sources of variance.

Consider an example of a two-factor design where factors A and B are manipulated. In a fixed effects model, the pooled within-cell variance is the only source of variation associated with subjects so $\hat{\sigma}^2 = MS_{s/Cells}$ in Equation 6. Because A and B are manipulated

$$\sum_{\text{Meas}} \sigma_{\text{Meas}}^2 = 0,$$

and generalized omega squared for factor A is

$$\hat{\omega}_{\rm G}^2 = \frac{SS_{\rm A} - df_{\rm A}MS_{\rm s/cells}}{(SS_{\rm A} - df_{\rm A}MS_{\rm s/cells}) + N \times MS_{\rm s/cells}},$$

which simplifies to partial omega squared

$$\hat{\omega}_{\rm G}^2 = \frac{SS_{\rm A} - df_{\rm A}MS_{\rm s/Cells}}{SS_{\rm A} + (N - df_{\rm A})MS_{\rm s/Cells}}$$

If factor A is a manipulated factor and factor b is a measured factor, then using Equation 7

$$\hat{\omega}_{G}^{2} = \frac{SS_{A} - df_{A}MS_{s/Cells}}{(SS_{A} - df_{A}MS_{s/Cells}) + (SS_{b} - df_{b}MS_{s/Cells})} + (SS_{Ab} - df_{Ab}MS_{s/Cells}) + N \times MS_{s/Cells}}$$

which simplifies to omega squared rather than partial omega squared

$$\hat{\omega}_{\rm G}^2 = \frac{SS_{\rm A} - df_{\rm A}MS_{\rm s/Cells}}{SS_{\rm T} + MS_{\rm s/Cells}}.$$

For the measured factor b,

$$\hat{\omega}_{G}^{2} = \frac{SS_{b} - df_{b}MS_{s/Cells}}{0(SS_{b} - df_{b}MS_{s/Cells}) + (SS_{b} - df_{b}MS_{s/Cells})} + (SS_{Ab} - df_{Ab}MS_{s/Cells}) + N \times MS_{s/Cells}}$$

which simplifies to

$$\hat{\omega}_{\rm G}^2 = \frac{SS_{\rm b} - df_{\rm b}MS_{\rm s/Cells}}{SS_{\rm T} + SS_{\rm A} + J \times MS_{\rm s/Cells}},$$

where J is the number of levels in factor A.

The generalized omega squared formulas (Equations 6 and 7) can be applied to higher order factorial designs. Table 2 provides formulas for omega squared in a three-factor fixed-effects design for various combinations of manipulated (uppercase letters) and measured (lowercase letters) factors. Keppel (1991, p. 438) provided an ANOVA table for a $3 \times 2 \times 2$ factorial design where the first two factors are manipulated, and the third factor is measured (grade level). For factor A

$$\hat{\omega}_{\rm P}^2 = \frac{33.63 - 2(1.75)}{33.63 + (60 - 2)1.75} = .223$$

Generalized omega squared for design ABc is computed as

$$\hat{\omega}_{\rm G}^2 = \frac{33.63 - 2(1.75)}{227.93 - 52.26 - 5.64 + (3 \times 2 - 2)1.75} = .170.$$

Partial omega squared is almost 24% greater than generalized omega squared. The omega effect sizes are

Table 2

Selected Formulas for	· Omega Squared	Statistics in a	Three-Factor	Analysis of Variance
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Source	Design	$\hat{\omega}_{G}^{2}$
α	ABC	$[SS_{\rm A} - df_{\rm A} \times MS_{\rm s/Cells}] / [SS_{\rm A} + (N - df_{\rm A})MS_{\rm s/Cells}]$
	ABc	$[SS_{\rm A} - df_{\rm A} \times MS_{\rm s/Cells}]/[SS_{\rm T} - SS_{\rm B} - SS_{\rm AB} + (JK - df_{\rm A})MS_{\rm s/Cells}]$
	AbC	$[SS_{\rm A} - df_{\rm A} \times MS_{\rm s/Cells}]/[SS_{\rm T} - SS_{\rm C} - SS_{\rm AC} + (JL - df_{\rm A})MS_{\rm s/Cells}]$
	Abc	$[SS_{\rm A} - df_{\rm A} \times MS_{\rm s/Cells}]/[SS_{\rm T} + MS_{\rm s/Cells}]$
	aBC	$[SS_{\rm a} - df_{\rm a} \times MS_{\rm s/Cells}]/[SS_{\rm T} - SS_{\rm B} - SS_{\rm C} - SS_{\rm BC} + KL \times MS_{\rm s/Cells}]$
	abC	$[SS_{\rm a} - df_{\rm a} \times MS_{\rm s/Cells}]/[SS_{\rm T} - SS_{\rm C} + L \times MS_{\rm s/Cells}]$
	aBc	$[SS_{\rm a} - df_{\rm a} \times MS_{\rm s/Cells}]/[SS_{\rm T} - SS_{\rm B} + K \times MS_{\rm s/Cells}]$
	abc	$[SS_{\rm a} - df_{\rm a} \times MS_{\rm s/Cells}] / [SS_{\rm T} + MS_{\rm s/Cells}]$
αβ	ABC	$[SS_{AB} - df_{AB} \times MS_{s/Cells}] / [SS_{AB} + (N - df_{AB})MS_{s/Cells}]$
	ABc	$[SS_{AB} - df_{AB} \times MS_{s/Cells}]/[SS_{T} - SS_{A} - SS_{B} + (JK - df_{AB})MS_{s/Cells}]$
	AbC	$[SS_{Ab} - df_{Ab} \times MS_{s/Cells}] / [SS_{T} - SS_{A} - SS_{C} - SS_{AC} + JL \times MS_{s/Cells}]$
	Abc	$[SS_{Ab} - df_{Ab} \times MS_{s/Cells}]/[SS_T - SS_A + J \times MS_{s/Cells}]$
	aBC	$[SS_{aB} - df_{Ab} \times MS_{s/Cells}]/[SS_{T} - SS_{B} - SS_{C} - SS_{BC} + KL \times MS_{s/Cells}]$
	abC	$[SS_{ab} - df_{ab} \times MS_{s/Cells}]/[SS_{T} - SS_{C} + L \times MS_{s/Cells}]$
	aBc	$[SS_{aB} - df_{aB} \times MS_{s/Cells}]/[SS_{T} - SS_{B} + K \times MS_{s/Cells}]$
	abc	$[SS_{ab} - df_{ab} \times MS_{s/Cells}]/[SS_{T} + MS_{s/Cells}]$
αβγ	ABC	$[SS_{ABC} - df_{ABC} \times MS_{s/Cells}]/[SS_{ABC} + (N - df_{ABC})MS_{s/Cells}]$
	ABc	$[SS_{ABc} - df_{ABc} \times MS_{s/Cells}]/[SS_{T} - SS_{A} - SS_{B} - SS_{AB} + JK \times MS_{s/Cells}]$
	AbC	$[SS_{AbC} - df_{AbC} \times MS_{s/Cells}] / [SS_{T} - SS_{A} - SS_{C} - SS_{AC} + JL \times MS_{s/Cells}]$
	Abc	$[SS_{Abc} - df_{Abc} \times MS_{s/Cells}]/[SS_{T} - SS_{A} + J \times MS_{s/Cells}]$
	aBC	$[SS_{\rm aBC} - df_{\rm aBC} \times MS_{\rm s/Cells}]/[SS_{\rm T} - SS_{\rm B} - SS_{\rm C} - SS_{\rm BC} + KL \times MS_{\rm s/Cells}]$
	abC	$[SS_{abC} - df_{abC} \times MS_{s/Cells}]/[SS_{T} - SS_{C} + L \times MS_{s/Cells}]$
	aBc	$[SS_{aBc} - df_{aBc} \times MS_{s/Cells}] / [SS_T - SS_B + K \times MS_{s/Cells}]$
	abc	$[SS_{\rm abc} - df_{\rm abc} \times MS_{\rm s/Cells}]/[SS_{\rm T} + MS_{\rm s/Cells}]$

Note. Uppercase letters indicate a manipulated factor; lowercase letters indicate a measured factor. Letters J, K, and L refer to the number of levels of factors A, B, and C, respectively.

smaller than the eta effect sizes computed earlier with the same data.

If a design contains random effects, Equation 6 can be used but the formulas for computing the variance components would be different than when all effects are fixed. Dodd and Schultz (1973) showed how to compute variance components when one or more of the factors are random.

Analysis of Covariance

If analysis of covariance is used and omega squared is the preferred effect size, Equation 7 can be used with $MS_{s/Cells}$ calculated as it is in the corresponding design without any covariates. That is,

$$MS_{\rm s/Cells} = \frac{\sum (n_{\rm Cell} - 1)S_{\rm Cell}^2}{\sum (n_{\rm Cell} - 1)},$$
 (8)

where summation is over all cells of the design, n_{Cell} is the sample size in a cell, and S_{Cell}^2 is the unadjusted variance in a cell. Alternatively, $MS_{s/Cells}$ can be obtained by pooling the sum of squares for the covariates and the sum of squares error and dividing by the pooled degrees of freedom for these sources. That is, $MS_{s/Cells} = (SS_{Covariate} + SS_{Error})/(N - J)$. As noted in the discussion following Equation 7, MS_{Error} would be the within-cell variance adjusted by the covariates.

The formulas for omega squared in Table 2 can be used for both two- or three-factor designs with one or more covariates. For three-factor designs with any combination of manipulated and measured factors, the formulas in Table 2 can be used noting that $MS_{s/Cells}$ is computed using Equation 8. For two-factor designs, the formulas in Table 2 are also appropriate with c representing the covariate, a measured factor.

Repeated Measures Design

In repeated measures designs, subjects are crossed with the repeated measures factors. Equation 6 can be used to calculate generalized omega squared for a repeated measures design.¹ The quantity $\hat{\sigma}^2$ can be estimated by the ratio of the pooled sum of squares for all sources of variance involving the subjects factor to the pooled degrees of freedom for these sources of variance. For example, in a single-factor repeated measures design having *J* levels, $\hat{\sigma}^2 = (SS_s + SS_{sA})/(N - J)$. The value of *N* is the total number of observations in the study N = Jn, where *n* is the number of individuals in the study. The estimate $\hat{\sigma}^2$ is the average of the variances in the *J* levels of the repeated measures factor (see Kirk, 1995, p. 254), so Equation 8 can also be used to calculate $\hat{\sigma}^2$. For a two-factor repeated measures design $\hat{\sigma}^2 = (SS_s + SS_{sA} + S_{sB} + SS_{sAB})/(N - JK)$. Equation 7 is more convenient to use for calculating generalized omega squared. The quantity $MS_{s/Cells}$ is $\hat{\sigma}^2$ and can be computed as just described. When Equation 7 is applied in a repeated measures design, MS_{Error} in $SS_{Effect} - df_{Effect} \times MS_{Error}$ is the mean square for the interaction of subjects and the effect. For example, in a two-way repeated measures design, MS_{Error} for the AB effect is MS_{sAB} . Typically, the terms involving Meas can be deleted because repeated measures factors are typically not measured factors.

Maxwell and Delaney (2000, p. 497) provided data for a 2 × 3 single-group repeated measures design. Partial omega squared for the three-level factor equals [289,920 - 2(3,560)]/[289,920 + (60 - 2)3,560] = .570. Generalized omega squared can be computed using the formula in Table 2 for effect α and design ABc where subjects is the c factor:

$$\hat{\omega}_{G}^{2} = \frac{289,920 - 2(3,560)}{1,133,940 - 285,660 - 105,120} = .364. + [(3)(2) - 2]8,393.33$$

¹ For a repeated measures design, assuming as typically would be true that the repeated measures factors are manipulated, the parameter defined in Equation 2 is

$$\frac{\sigma_{\rm Effect}^2}{\delta \times \sigma_{\rm Effect}^2 + \sum_{\lambda} \sigma_{\lambda}^2},$$

where λ ranges over all sources of variance that involve subjects. Thus in a one-factor repeated measures design, the denominator would include σ_e^2 , σ_{sA}^2 , and σ_s^2 , and the parameter would equal

$$\frac{\sigma_A^2}{\sigma_A^2 + \sigma_e^2 + \sigma_s^2 + \sigma_{sA}^2}.$$

It can be shown that when Equation 6 is applied in a repeated measures design, the denominator underestimates correct denominator. For example, for a one-factor repeated measures design, the expected value of the denominator of Equation 6 is

$$\sigma_{\rm A}^2 + \sigma_{\rm e}^2 + \sigma_{\rm s}^2 + \left(\frac{J-1}{J}\right)\sigma_{\rm sA}^2$$

(

An alternative to Equation 6, on the basis of results in Dodd and Schultz (1973) is

Source	Design	$\hat{\omega}_{G}^{2}$
α	AB	$[SS_{\rm A} - df_{\rm A} \times MS_{\rm s/A}]/[SS_{\rm A} + (\tilde{N} - df_{\rm A} \times MS_{\rm s/A} + \tilde{\rm N}(K-1) \times MS_{\rm Bs/A}]$
	aB	$[SS_{a} - df_{a} \times MS_{s/a}]/[SS_{T} - SS_{B} + MS_{s/a} + (K - 1) \times MS_{Bs/a}]$
	ACB	$[SS_{A} - df_{A} \times MS_{s/AC}]/[SS_{A} + (\tilde{N} - df_{A}) \times MS_{s/AC} + \tilde{N}(K - 1)MS_{Bs/AC}]$
	aCB	$[SS_{a} - df_{a} \times MS_{s/aC}]/[SS_{T} - SS_{B} - SS_{C} - SS_{BC} + L \times MS_{s/aC} + L(K - 1) \times MS_{Bs/AC}]$
	AcB	$[SS_{\rm A} - df_{\rm A} \times MS_{\rm s/ac}]/[SS_{\rm T} - SS_{\rm B} - SS_{\rm AB} + MS_{\rm s/ac} + J(K-1) \times MS_{\rm Bs/Ac}]$
	acB	$[SS_{\rm a} - df_{\rm a} \times MS_{\rm s/ac}]/[SS_{\rm T} - SS_{\rm B} + MS_{\rm s/ac} + (K - 1) \times MS_{\rm Bs/ac}]$
αγ	ACB	$[SS_{\rm AC} - df_{\rm AC} \times MS_{s/\rm AC}]/[SS_{\rm AC} + (\tilde{\rm N} - df_{\rm AC}) \times MS_{s/\rm AC} + \tilde{\rm N}(K-1) \times MSB_{s/\rm AC}]$
	aCB	$[SS_{\rm aC} - df_{\rm aC} \times MS_{\rm s/aC}]/[SS_{\rm T} - SS_{\rm B} - SS_{\rm C} - SS_{\rm BC} + L \times MS_{\rm s/aC} + L(K - 1) \times MSB_{\rm s/aC}]$
	AcB	$[SS_{Ac} - df_{Ac} \times MS_{s/Ac}]/[SS_{T} - SS_{A} - SS_{B} - SS_{AB} + J \times MS_{s/Ac} + J(K - 1) \times MS_{Bs/Ac}]$
	acB	$[SS_{\rm ac} - df_{\rm ac} \times MS_{\rm s/ac}]/[SS_{\rm T} - SS_{\rm B} + MS_{\rm s/ac} + (K - 1) \times MS_{\rm Bs/ac}]$

Selected Formulas for Generalized Omega Squared Statistics for Between-Subjects Factors in Mixed-Model Designs

Note. A and C are between-subjects factors, and B is a repeated measures factor. \tilde{N} is the total number of individuals in the study.

Mixed Design

Equation 6 or 7 can be used to calculate generalized omega squared for a mixed design. In Equation 6, $\hat{\sigma}^2$ is computed by pooling all sources of variance involving subjects. For example, in a design with one between-subjects factor and one repeated-measures factor, $\hat{\sigma}^2 = (SS_{s/A} + SS_{Bs/A})/(df_{s/A} + df_{Bs/A}) = MS_{s/Cells}$. This formula for $\hat{\sigma}^2$ is equivalent to pooling the variance over the cells formed by combinations of the between-subjects and repeated measures factors. As an example, consider a design in which factors A and B are both manipulated. Because A and B are both manipulated

$$\frac{\hat{\sigma}_{\text{Effect}}^2}{\delta \times \hat{\sigma}_{\text{Effect}}^2 + \sum_{\nu} \hat{\sigma}_{\nu}^2 + \sum_{\lambda} \frac{\pi_{\lambda}}{N} MS_{\lambda}},$$

where π_{λ} is the product of the number of levels of the factors defining mean square for the λ effect. Thus for MS_{sA} , $\pi_{\lambda} = nJ$. It can be shown that when the preceding equation is applied in a repeated measures design, the denominator overestimates the correct denominator. For example, for a one-factor repeated measures design the expected value of the denominator of equation would be

$$\sigma_{\rm A}^2 + \left(\frac{J+1}{J}\right)\sigma_{\rm e}^2 + \sigma_{\rm s}^2 + \sigma_{\rm sA}^2 \,.$$

We did not select Equation 6 because we believe it is the better way to estimate the denominator of generalized omega squared. Rather, we selected it because it can be applied to all designs considered in this article. The alternative to Equation 6 does not apply to designs with covariates.

$$\sum_{\text{Meas}} (SS_{\text{Meas}} - df_{\text{Meas}} \times MS_{\text{M}_\text{Error}}) = 0.$$

Then Equation 7 becomes

$$\hat{\omega}_{\rm G}^2 = \frac{SS_{\rm Effect} - df_{\rm Effect} \times MS_{\rm Error}}{\delta(SS_{\rm Effect} - df_{\rm Effect} \times MS_{\rm Error}) + N \times MS_{\rm s/Cells}}$$

If the A factor is of interest, we have

$$\hat{\omega}_{\rm G}^2 = \frac{SS_{\rm A} - df_{\rm A}MS_{\rm s/A}}{(SS_{\rm A} - df_{\rm A}MS_{\rm s/A}) + N \times MS_{\rm s/Cells}}$$

where *N* is the total number of observations in the data set.

Tables 3 and 4 provide the formulas derived from Equation 7 for computing generalized omega squared for selected between-subject and repeated measures factors. In these tables \tilde{N} is the number of individuals in the study.

Kirk (1995, p. 527) provided an ANOVA summary table for a $2 \times 2 \times 4$ mixed-model design with the last factor being within-subjects. Assuming an AcB design, partial omega squared for factor A would equal [3.125 - 1(1.812)]/[3.125 + (8 - 1)1.812] = .083. By contrast,

$$\hat{\omega}_{\rm G}^2 = \frac{3.125 - 1(1.812)}{235.5 - 194.5 - 19.375} = .054.$$
$$+ 1.812 + 2(2 - 1)0.396$$

Randomized Block Design

Kirk (1995, pp. 293–298) distinguished between a randomized block design (RBD) and a generalized randomized block design (GRBD). In an RBD the

Table 3

Source	Design	$\hat{\omega}_{G}^{2}$
α	AB	$[SS_{\rm B} - df_{\rm B} \times MS_{\rm Bs/A}]/[SS_{\rm B} + \tilde{N} \times MS_{\rm s/A} + (\tilde{N} - 1)(K - 1) \times MS_{\rm Bs/A}]$
	aB	$[SS_{\rm B} - df_{\rm B} \times MS_{\rm Bs/A}]/[SS_{\rm T} + MS_{\rm s/A}]$
	ACB	$[SS_{\rm B} - df_{\rm B} \times MS_{\rm Bs/AC}]/[SS_{\rm B} + \tilde{N} \times MS_{\rm s/AC} + (\tilde{N} - 1)(K - 1) \times MS_{\rm Bs/AC}]$
	aCB	$[SS_{\rm B} - df_{\rm B} \times MS_{\rm Bs/aC}]/[SS_{\rm T} - SS_{\rm C} - SS_{\rm CB} + L \times MS_{\rm s/aC} + (L-1)(K-1) \times MS_{\rm Bs/aC}]$
	AcB	$[SS_{\rm B} - df_{\rm B} \times MS_{\rm Bs/Ac}]/[SS_{\rm T} - SS_{\rm A} - SS_{\rm AB} + J \times MS_{\rm s/ac} + (L-1)(K-1) \times MS_{\rm Bs/Ac}]$
	acB	$[SS_{\rm B} - df_{\rm B} \times MS_{\rm Bs/ac}]/[SS_{\rm T} + MS_{\rm s/ac}]$
αβ	AB	$[SS_{\rm AB} - df_{\rm AB} \times MS_{\rm Bs/A}]/[SS_{\rm AB} + \tilde{N} \times MS_{\rm s/A} + (\tilde{N} - df_{\rm A}) (K - 1) \times MSB_{\rm Bs/A}]$
	aB	$[SS_{aB} - df_{aB} \times MS_{Bs/a}]/[SS_{T} + \tilde{N} \times MS_{s/a} + (K - 1) \times MS_{Bs/a}]$
	ACB	$[SS_{AB} - df_{AB} \times MS_{Bs/AC}]/[SS_{AB} + \tilde{N} \times MS_{s/AC} + (\tilde{N} - df_{A}) (K - 1) \times MS_{Bs/AC}]$
	aCB	$[SS_{\rm aB} - df_{\rm aB} \times MS_{\rm Bs/aC}] / [SS_{\rm T} - SS_{\rm B} - SS_{\rm C} - SS_{BC} + L \times MS_{\rm s/aC} + L(K-1) \times MS_{\rm Bs/aC}]$
	AcB	$[SS_{AB} - df_{AB} \times MS_{Bs/Ac}]/[SS_{T} - SS_{A} - SS_{B} + J \times MS_{s/Ac} + (K - 1) \times MS_{Bs/Ac}]$
	acB	$[SS_{\rm aB} - df_{\rm aB} \times MS_{\rm Bs/ac}]/[SS_{\rm T} - SS_{\rm B} + MS_{\rm s/ac} + (K - 1) \times MS_{\rm Bs/ac}]$
αγβ	ACB	$[SS_{ACB} - df_{ACB} \times MS_{Bs/AC}]/[SS_{ACB} + \tilde{N} \times MS_{s/AC} + (\tilde{N} - df_{A})(K - 1) \times MS_{Bs/AC}]$
	aCB	$[SS_{aCB} - df_{aCB} \times MS_{Bs/aC}]/[SS_T - SS_B - SS_C - SS_{BC} + L \times MS_{s/aC} + L(K - 1) \times MS_{Bs/aC}]$
	AcB	$[SS_{ACB} - df_{ACB} \times MS_{Bs/AC}]/[SS_{T} - SS_{A} - SS_{B} + J \times MS_{s/AC} + (K - 1) \times MS_{Bs/AC}]$
	acB	$[SS_{acB} - df_{acB} \times MS_{Bs/ac}]/[SS_{T} - SS_{B} - MS_{s/ac} + (K - I) \times MS_{Bs/ac}]$

Selected Formulas for Generalized Omega Squared Statistics for Within-Subjects Factors in Mixed-Model Designs

Note. A and C are between-subjects factors, and B is a repeated measures factor. \tilde{N} is the total number of individuals in the study.

number of participants within a block is equal to the number of treatments to which the participants will be assigned. In a GRBD the number of participants in a block is a multiple of the number of treatments to which the participants will be assigned. For example, suppose a control treatment and two treatments designed to reduce fear of spiders are to be compared. Prior to assignment to treatments, researchers obtain measurements of fear of spiders on 60 participants. If the participants are ranked, placed into blocks of three, and assigned from within blocks to the treatments, the design is an RBD. If a median split is used so that participants are placed in two blocks of 30 each and then assigned to treatments, the design is a GRBD. For an RBD, generalized omega squared can be computed by using the procedures for a repeated measures design, with blocks replacing participants. It is also possible for blocks to be assigned to levels of one or more between-subjects factors. Then generalized omega squared can be computed by using the procedures for a mixed design, with blocks replacing participants. For a GRBD, generalized omega squared can be computed by using the procedures for a betweensubjects design. If in addition there are repeated measures factors, generalized omega squared can be computed by using the procedures for a mixed design.

Table 4

Discussion

Providing a measure of an effect size is now being required by the editors of several prominent journals in education and psychology to enhance the meaningfulness of the results when statistical hypothesis tests are used. The current practice of reporting the partial eta or partial omega squared statistic to estimate an effect size for each factor in multifactor research design is often misleading and inappropriate. Routine use of these measures of effect size does not appropriately consider the design features of a research study when estimating the magnitude of an effect. In the case of measures of association in an ANOVA context, little attention has been given to the design features, which can affect the size of the estimated effects. Textbooks and computer output often imply that the partial eta or partial omega squared statistics are the appropriate effect-size measures to use in all research designs that include more than a single explanatory variable. This is an appropriate recommendation if the researcher manipulates all of the factors in the design. However, if a research design includes one or more measured factors, partial eta or omega squared statistics often provide an undesirable effectsize measure. Researchers often include one or more measured factors as covariates or as blocking variables to increase the statistical power of their hypothesis tests. These factors increase the statistical power by reducing the error variance. However, although power of the analysis is increased, the interpretation of the hypothesis test is unchanged. The reduced error variance as a result of blocking or covarying, however, does affect the interpretation of effect-size measures. Reducing the error variance restricts the population for whom the effect is being estimated.

Partial eta and omega squared also can provide indices that are inappropriately compared to Cohen's (1988) guidelines for defining the small, medium, and large effects. In suggesting the guidelines, Cohen cited studies that compared unrestricted populations (e.g., differences between men and women). Cohen used differences between men and women on several variables to help define $.2\sigma$ as a small difference. He did not restrict these populations, for example by using age as a covariate or a blocking variable. To do so would have changed the magnitude of the difference that today is considered small. If researchers continue to use the same guidelines to define small, medium, and large effects across a variety of research designs, then it seems reasonable to expect researchers to provide effect-size estimates relative to the full range of the population rather than a restricted population. Consequently, the use of partial eta or partial omega squared in many situations diminishes the usefulness of the new editorial policy.

The generalized eta and omega squared statistics have two major advantages. First, these statistics provide measures of effect size that are comparable across a wide variety of research designs that are popular in education and psychology. Second, these effect-size measures provide indices of effect that are consistent with Cohen's (1988) guidelines for defining the magnitude of the effect. Cohen pointed out three decades ago that design considerations must be used when computing an effect-size measure. Currently, most researchers who choose to report an effect size as the proportion of variance explained have ignored Cohen's caution. Using the procedures outlined in the present article, researchers can correct this omission and provide more comparable effect-size measures. A final cautionary note should be restated. Generalized eta and omega squared statistics can provide comparable effect-size measures for studies that use different outcome measures and different research designs when studying a common target population. If the target populations studied differ substantially with respect to their variances, effect-size measures would not be comparable. Professional judgment must be used when determining when it is reasonable to compare effect sizes across a series of studies.

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Received September 17, 2001 Revision received April 11, 2003

Accepted June 23, 2003 ■

Call for Nominations: Rehabilitation Psychology

The APA Publications and Communications (P&C) Board has opened nominations for the editorship of *Rehabilitation Psychology* for the years 2006–2011. Bruce Caplan, PhD, is the incumbent editor.

Candidates should be members of APA and should be available to start receiving manuscripts in early 2005 to prepare for issues published in 2006. Please note that the P&C Board encourages participation by members of underrepresented groups in the publication process and would particularly welcome such nominees. Self-nominations are also encouraged.

Rehabilitation Psychology will transition from a division publication to an "all APA" journal in 2006, and the successful candidate will be involved in making suggestions to the P&C Board and APA Journals staff about the transition process.

Gary R. VandenBos, PhD, and Mark Appelbaum, PhD, have been appointed as cochairs for this search.

To nominate candidates, prepare a statement of one page or less in support of each candidate. Address all nominations to

> Rehabilitation Psychology Search Committee Karen Sellman, Search Liaison Room 2004 American Psychological Association 750 First Street, NE Washington, DC 20002-4242

The first review of nominations will begin December 8, 2003. The deadline for accepting nominations is December 15, 2003.